Exercise 12

Solve the differential equation.

$$\frac{d^2R}{dt^2} + 6\frac{dR}{dt} + 34R = 0$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $R = e^{rt}$.

$$R = e^{rt} \quad \rightarrow \quad \frac{dR}{dt} = re^{rt} \quad \rightarrow \quad \frac{d^2R}{dt^2} = r^2e^{rt}$$

Plug these formulas into the ODE.

$$r^2e^{rt} + 6(re^{rt}) + 34(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^2 + 6r + 34 = 0$$

Solve for r.

$$r = \frac{-6 \pm \sqrt{36 - 4(1)(34)}}{2} = \frac{-6 \pm \sqrt{-100}}{2} = -3 \pm 5i$$

$$r = \{-3 - 5i, -3 + 5i\}$$

Two solutions to the ODE are $e^{(-3-5i)t}$ and $e^{(-3+5i)t}$. By the principle of superposition, then,

$$\begin{aligned}
R(t) &= C_1e^{(-3-5i)t} + C_2e^{(-3+5i)t} \\
&= C_1e^{-3t}e^{-5it} + C_2e^{-3t}e^{5it} \\
&= e^{-3t}(C_1e^{-5it} + C_2e^{5it}) \\
&= e^{-3t}[C_1(\cos 5t - i\sin 5t) + C_2(\cos 5t + i\sin 5t)] \\
&= e^{-3t}[(C_1 + C_2)\cos 5t + (-iC_1 + iC_2)\sin 5t] \\
&= e^{-3t}(C_3\cos 5t + C_4\sin 5t),
\end{aligned}$$

where C_1, C_2, C_3, and C_4 are arbitrary constants.